

FAQ's

re:

Mean Field Theory  
for Chaotic Regime

# FAQ's re: Calculations for Chaos

→ what does  $F$  mean?

$$\frac{dr}{dz} = \frac{B_r}{B_T}, \quad r d\theta = \frac{B_\theta}{B_T} \quad (B_T \text{ uniform})$$

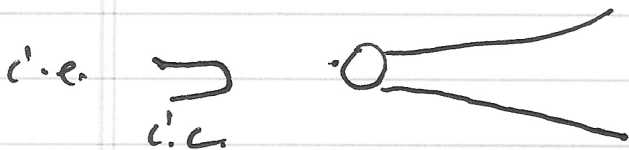
define Hamiltonian orbits for lines.

Then  $F$  is Liouvilian function for orbits,  
and  $\underline{B} \cdot \underline{\nabla} F = 0 \rightarrow$  Liouville Egn.

→ seek?  $\rightarrow \langle F \rangle$ ,  $\langle F \rangle = \langle f(z, r) \rangle$   
mean

begs: scales? meaning of brackets?

-  $\langle F \rangle$  is mean pdf. Idea is to calculate evolution of pdf as lines wander.



- implicit is 2 scale picture / ansatz.

i.e.  $\tilde{B}_r = \sum_{m,n} B_{m,n}(r) \exp[i(m\theta - n\phi)]$

$k_0 = m/r$   
 $k_z = n/R$

} Flat scales

$\langle f(z) \rangle \rightarrow$  slow scale

$\equiv$   $k_z \gg \left( \frac{1}{\langle f \rangle} \frac{\partial \langle f \rangle}{\partial z} \right)$

and thus:

$\langle \rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi}$

$\rightarrow$  evolution equation:

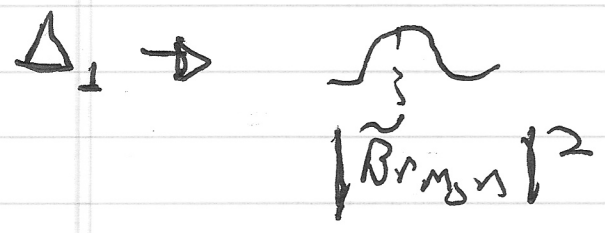
$\frac{\partial \langle f \rangle}{\partial z} = - \frac{\partial}{\partial r} \left\langle \sum_{m,n} \frac{\tilde{B}_{r-m}}{B_r} \mathcal{F}_{m,n} \right\rangle$   
 $\equiv - \frac{\partial}{\partial r} \left[ \text{radial flux of pdf.} \right]$

- issues: - hierarchy truncation?  
 - closure?

- use linear response iteration  
 - near field theory  
 - quasi-linear theory

why? →  $k_u < 1$

$$k_u = \frac{\rho_{ac}}{\Delta_L} \frac{\tilde{B}_r}{B_0}$$



→ radial scale of scattering field

$\rho_{ac} \rightarrow \frac{1}{\Delta_{\text{min}}}$  → inverse spatial bandwidth of  $k_u$

$k_u < 1 \rightarrow$  weak scattering, many kicks  
 ⇒ diffusion

$k_u > 1 \rightarrow$  strong scattering, QLT, not ok.

Calculation of  $\delta F \rightarrow$  Linearize!

$$\underline{B}_0 \cdot \nabla \delta F = -\tilde{B}_r \frac{\partial \langle F \rangle}{\partial r}$$

$$\delta F \Big|_{\underline{B}} \sim e^{i(k_y y - k_z z)}$$

$$y = v t$$

n.b.  $k_z \rightarrow k_z + i\epsilon$ , so  $\left\{ \begin{array}{l} \delta F \rightarrow 0 \Leftrightarrow z \rightarrow -\infty \\ \text{causality} \end{array} \right.$

$$\delta F_k = \frac{e}{\left[ k_z - k_y \frac{\langle B_\theta \rangle}{B_T} + i\epsilon \right]} \left( -\tilde{B}_r \frac{\partial \langle F \rangle}{\partial r} \right)$$

$\therefore$

$$\rightarrow \Gamma = -D_M \frac{\partial \langle F \rangle}{\partial r}$$

$$\Rightarrow D_M = \sum_k \left| \frac{\tilde{B}_r}{B_0} u \right|^2 \pi \delta \left( k_z - k_y \frac{\langle B_\theta \rangle}{B_T} \right)$$

$\downarrow$   
 radial diffusion equation for  $\langle F \rangle$ , due  
 field wandering

→ why  $D_m$  ? → trajectory can hop either direction

→ Origin of irreversibility? → island overlap.



{ Field line scatters/wanders from resonance to resonance

→  $D_m$  → random walk : { step ?  
time/parallel step ?  
increment. }

$$dr \equiv \text{radial step} = l_{ac} \frac{\tilde{B}_r}{B_0}$$

$$\text{parallel step} = l_{ac}$$

$$D \sim l_{ac}^2 \frac{|\frac{\tilde{B}_r}{B_0}|^2}{l_{ac}} \sim l_{ac} \left| \frac{\tilde{B}_r}{B_0} \right|^2$$

→ How measure decorrelation length (i.e. length over which orbit scatters from its unperturbed trajectory) ?

~ FOM

$$\overline{k_0^2 r^2 \langle d\theta^2 \rangle} \sim 1$$

$$\Rightarrow m^2 \langle d\theta^2 \rangle \sim 1$$

$$\frac{d\theta}{dz} = \frac{\langle B_0 \rangle}{r B_z}$$

$$\langle B_0 \rangle = \langle B_0(r) \rangle$$

so

$$\frac{d(\theta_0 + d\theta)}{dz} = \frac{\langle B_0(r_0) \rangle}{r_0 B_z} + \frac{dr}{dz} \left( \frac{\langle B_0 \rangle}{r B_z} \right)'_{r_0}$$

$$d\theta \sim \int_{z_0}^z dz' \frac{dr}{R} \left( \frac{1}{r g(r)} \right)'$$

$$= \int_{z_0}^z dz' \frac{dr}{R} \left( \frac{1}{R g(r)} \right)'$$

$$\langle d\theta^2 \rangle \sim O_M Z^3 \left[ \left( \frac{1}{R g} \right)' \right]^2$$

$$\overline{m^2 \langle d\theta^2 \rangle} \sim 1 \sim \overline{m^2} O_M Z^3 \frac{Z^{1/2}}{R^2 (g^2)^2}$$

$$\Rightarrow l_c \sim \left[ \kappa \Omega^2 D_M \left( \frac{r^2 v^2}{g^2} \right) \left( \frac{1}{R_g} \right)^2 \right]^{-1/3} L$$

→ How does  $l_c$  relate to Lyapunov exponent



$h \equiv$  rate for exponential divergence of orbits

$l_c \equiv$  rate for exponential divergence from linear orbit.

$$\Rightarrow h \sim l_c$$